Behavior strategies

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Summary of the slides

1. Information set
2. Behavior strategy
3. Equivalence of strategies
4. Player without memory
5. Absent minded player
6. Game of perfect recall

Behavior strategy

Definition

A behavior strategy for a player in an extensive form game is a map defined on the collection of her information sets and providing to each information set a probability distribution over the actions at the information set

Remembering the formal definition of information set for player i:

Definition

An information set for a player i is a pair $(U_i, A_i(U_i))$ with the following properties:

1. $U_i \subset P_i$ *is a nonempty set of vertices* $v_1, \dots, v_k$
2. *each* $v_j \in U_i$ *has the same number of children*
3. $A_i(U_i)$ *is a partition of the children of* $v_1 \cup \cdots \cup v_k$ *with the property that each element of the partition contains exactly one child of each vertex* v_j

Reference example

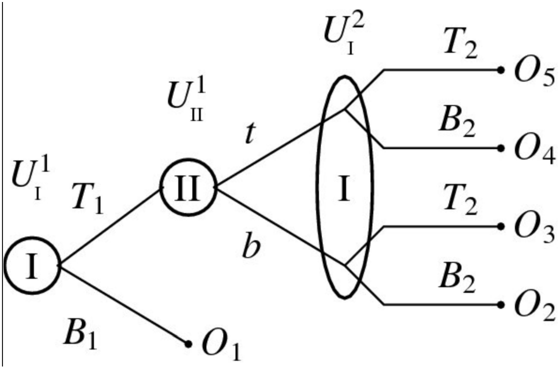


Figure: A simple tree

For Player I a behavior strategy is $(p, 1-p), (q, 1-q)$, where p represents the probability attached to choice B_1, q to B_2.

Mixed versus behavior

The pure strategies for Player 1 are 4, the set of mixed strategies is Σ_4, geometrically a three dimensional subset of $\mathbb{R}^4$

The set of behavior strategies is geometrically a square in $\mathbb{R}^2$.

Behavior strategies are more natural, especially in large games.

Equivalence of strategies

A key issue:

Do behavior strategies and mixed strategies can be indifferently used?

Formalizing: denote by $p(x;\sigma)$ the probability that a vertex x is reached under the strategy σ

Definition

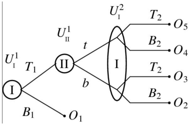
The strategies mixed/behavior σ_i and b_i are equivalent for Player I if for every strategy σ_{-i}[1] of the other players it holds

$$p(x;\sigma_i,\sigma_{-i}) = p(x;b_i,\sigma_{-i}).$$

For the equivalence it is enough to check on the leaves

[1] σ_{-i} can be formed by either behavior or mixed strategies

An example of equivalence



Set of pure strategies for the first player: $\{B_1B_2, B_1T_2, T_1B_2, T_1T_2\}$, suppose second player plays $(q, 1-q)$. Find a behavioral strategy equivalent to the mixed $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$ for the first. Probability induced on the terminal nodes by $\left[(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0), (q, 1-q)\right]$:

$$\frac{2}{3} : O_1 \qquad \frac{1}{3}q : O_2 \qquad 0 : O_3 \qquad \frac{1}{3}(1-q) : O_4 \qquad 0 : O_5$$

Call α the probability to select B_1, β the probability to select B_2. The probabilities at the terminal nodes become:

$$\alpha : O_1 \qquad (1-\alpha)q\beta : O_2 \qquad (1-\alpha)q(1-\beta) : O_3 \qquad (1-\alpha)(1-q)\beta : O_4 \qquad (1-\alpha)(1-q)(1-\beta) : O_5$$

Easy to see that $\alpha = \frac{2}{3}$, $\beta = 1$ does the job.

Why equivalence

Theorem

Let $s = (s_1, \dots, s_n)$ be a mixed strategies equilibrium profile. Let b_i be a behavior strategy for Player i equivalent to s_i. Then for every Player j it is $u_j(s) = u_j(b)$, where $b = (b_i, s_{-i})$.

If a player changesa strategy with an equivalent one the utilities remain the same for all players.

Mixed not behavior

Consider the following (strange) game

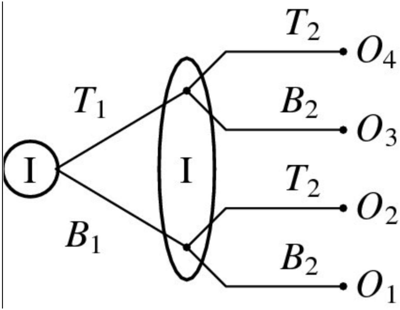


Figure: A player without memory

Consider the mixed strategy $(\frac{1}{2}, 0, 0, \frac{1}{2})$. Doest this have an equivalent behavior strategy?

Cont'd

Suppose behavior is $(p, 1-p), (q, 1-q)$. Then it must be

$$pq = \frac{1}{2} \tag{1}$$

$$p(1-q) = 0 \tag{2}$$

$$(1-p)q = 0 \tag{3}$$

$$(1-p)(1-q) = \frac{1}{2} \tag{4}$$

(2), (3) cannot hold at the same time. Thus not always a mixed has an equivalent behavior strategy.

Behavior not mixed

Consider the following (strange) game

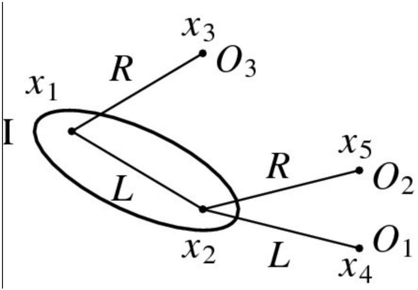


Figure: An absent minded driver

Vertex O_2 represents home. Can an absent minded driver join it with a mixed strategy? What if he uses behavior strategy?

From behavior to mixed

Theorem

Suppose Γ is an extensive form game such that every vertex which is not a leaf has at least two children. Then every behavior strategy of player i has an equivalent mixed strategy if and only if each information set of player i intersect every path starting at the root at most once.

The fact that the condition is necessary is not difficult to prove, the absent minded player provides the idea how to prove it.

Much more involved is to prove that the condition is also sufficient

From behavior to mixed

Definition

Player i has perfect recall if

1. *Every path from the root to a leaf intersects every information set of player i at most once*
2. *Every two paths from the root ending in the same information set pass through the same information sets of i and in the same order; moreover in each information set the two paths take the same action*

A game is called a game with perfect recall if each player has a perfect recall

A game without perfect recall

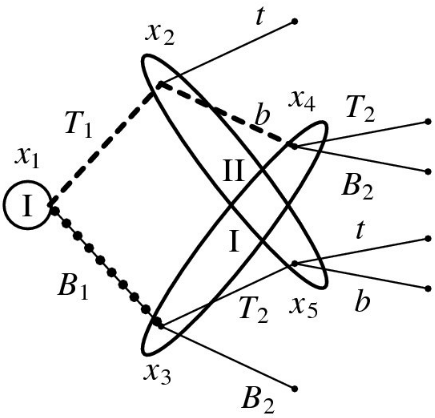


Figure: The first player does not have perfect recall

Another example

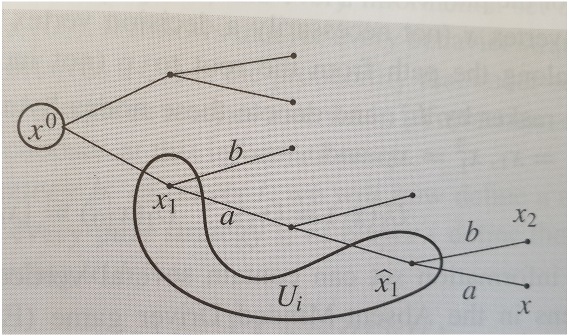


Figure: The player does not have perfect recall since she passes twice through the same information set

The second theorem

Theorem

Let Γ be a game. If player i has perfect recall, then every mixed strategy for player i has an equivalent behavior strategy

Corollary

In a game of perfect recall the equilibria with mixed strategies and behavior strategies are the same